B Probability Distributions

This appendix summarizes several different families of probability distributions relevant to the topics introduced in this book.¹ The distributions are represented by either probability mass functions or probability density functions, and the relevant functions are provided along with the parameters that govern each distribution. Plots show how the various parameters influence the distribution. The index includes page references to where these distributions are used in the body of the book. Some distributions are univariate, meaning they are distributions over a scalar variable; others are multivariate, meaning they are distributions over multiple variables.

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameters</th>
<th>Distribution Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>( a ) lower bound</td>
<td>( p(x) = \frac{1}{b-a} ) with ( x \in (a, b) )</td>
</tr>
<tr>
<td>( U(a, b) )</td>
<td>( b ) upper bound</td>
<td></td>
</tr>
</tbody>
</table>

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \] with \( x \in \mathbb{R} \)

## Appendix B. Probability Distributions

### Beta Distribution

**Beta** \((\alpha, \beta)\) with \(\alpha > 0\) and \(\beta > 0\) are shape parameters.

The probability density function is given by:

\[
p(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}
\]

with \(x \in (0, 1)\).

![Beta Distribution Diagram](image)

### Gaussian Distribution

**Gaussian** (multivariate) \(\mathcal{N}(\mu, \Sigma)\) with \(\mu\) mean and \(\Sigma\) covariance.

The probability density function is given by:

\[
p(x) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x - \mu)^\top \Sigma^{-1} (x - \mu)\right)
\]

where \(n = \text{dim}(x)\) and \(x \in \mathbb{R}^n\).

![Gaussian Distribution Diagram](image)

### Dirichlet Distribution

**Dirichlet** \(\text{Dir}(\alpha)\) with \(\alpha > 0\) concentration parameters.

The probability density function is given by:

\[
p(x) = \frac{\Gamma(\alpha_0)}{\prod_{i=1}^n \Gamma(\alpha_i)} \prod_{i=1}^n x_i^{\alpha_i-1}
\]

where \(\alpha_0 = \sum_i \alpha_i\), \(x_i \in (0, 1)\) and \(\sum_i x_i = 1\).

![Dirichlet Distribution Diagram](image)